SME M1
1/10   1/9   1/8
Practice Problems

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Sketch your graphs freehand. Don't use a ruler unless it's absolutely necessary. Use blank (no lines) paper if you have it. We are not trying to produce perfect graphs: we just want graphs that contain the essential ideas about a curve. Don't include details that are not important.

Make sure that you do the following:
- Plot the axes and label them.
- Label all the important features (lines, curves, points of intersection, etc.)
- Label and explain graph operations (shift, rotate, etc.)
- Give equations for your curves, and for each operation step.

1   Constant curves and points

1. On the same pair of axes, plot $y = 1$, $y = 5$ and $y = -3$.

2. Plot $x = -2$, $x = -1$ and $x = 5$ on the same axes.

3. Plot the point $P(1,3)$ and show the constant lines. Give an interpretation of the symbol $(1,3)$ in English.

4. Plot the point $Q(-3,-2)$ and give an interpretation in English. Show all important features of this graph, including the constant curves, axes, labels etc.

2   Shift operations

5. Start with $y = 5$ and plot $y = 5 + 1$ by doing a shift operation.

6. Start with $y = -2$ and do a shift operatio to get the graph of $y = -2 - 4 = -6$.

7. Do a shift on $x = a$, $a > 0$, to get the graph of $x = a - 2$.

8. Do a shift on $x = -a$, $a > 0$ to get the graph of $x = -a + 1$.

9. Start with $y = a$, $a > 0$, and do a LHS shift to get the graph of $y + 1 = a$.

10. Start with $x = -3$ and do a LHS shift to get the graph of $x - 4 = -3$.

11. Begin with $y = 5$ do a LHS shift and a RHS shift to get the graph of $y + 1 = 5 + 1$. Show both shift operations in your drawing.

12. Begin with $x = -5$. Do two shift operations: a LHS shift and a RHS shift to get $x + 2 = 5 - 1$. Show both operations in your graph drawing.

3   Stretch and shrink

13. Start with $y = 5$ and $x = 5$. Do stretch operations by multiplying the right-hand sides by 2. Label the operations. Show what they do.

14. Start with $y = -6$ and $x = -6$. Draw them on the same axes. Do shrink operations by multiplying the right-hand sides by 1/3. Label the operations and show what they do.

15. Reflect $x = 2$ and $y = -2$ by multiplying the left-hand-side by -1. Show the effects of the operations on your graph.

16. Fill in the table. Start with curve $y = a$ where a is a positive, $a > 0$. How can we get the desired result?

Curve	Operations on $y = a$
$y + 1 = a$	
$y = 2a - 1$	Stretch by 2, then shift down by 1.
$y = 2(a + 1)$	
$y = (2a + 5)/3$	

17. Start with $y = a$. How can we get the desired curve? What operations must we do?

Curve	Operations on $y = a$
$\dfrac{y}{3} - 1 = a$	Stretch by 3 then shift up by 1.
$y + 2 = a$	
$3y - 1 = a$	
$\dfrac{1}{2}y - 2 = a$	

18. Start with curve $x = a$ where a is a positive. How can we get the desired result? Fill in the table.

Curve	Operations on $x = a$
$x + 1 = a$	
$x = 2a - 1$	
$x = 2(a + 1)$	Shift right by 1, then stretch by 2.
$x = \dfrac{2a + 1}{3}$	

19. Start with curve $x = a$ where a is a positive. How can we get the desired result? Fill in the table.

Curve	Operations on $x = a$
$x + 2 = a$	
$3x - 1 = a$	
$\frac{1}{2}x - 2 = a$	
$2(x + 1) = a$	Shift left by 1, then shrink to 1/2.

4   Reflection

20. For $a > 0$, plot $y = a$, $x = a$ and their reflections $y = -a$, $x = -a$.

21. For positive constant a, plot $y = -4a$ by doing operations. Start with $y = a$. First do a stretch by 4, then do a reflection.

22. Let $a > 0$. Plot $x = -(a + 1)$. Start with $x = a$, do a shift, then a reflection. Label everything and show the effect of the operations.

23. Plot $y = -(a - 1)/2$ by doing three operations: shift, shrink, reflect. Start with the basic curve $y = a$, with positive a.

24. Start with $x = 5$. Plot $-2(x + 1) = 5$ by doing three operations: shift, shrink and then reflect.

5   Rotations

25. Start with $y = 5$. Do a reflection. Then rotate by $90°$. Then reflect. Then rotate by $-90°$. Draw all the steps and show the result. Label everything. Don't forget to label the axes and the operations that you are doing.

26. Start with $x = -5$. Do a $-90°$ rotation. Then do a reflection. Then do a $90°$ rotation. Draw all steps and label everything.

27. Start with $y = 1$. Do a shift up by 1. Reflect. Stretch by 2. Rotate by $90°$. What do you get? Show all the steps and label everything.

6   Curves $y = x$, $y = |x|$ and $y = x^2$

Use the special square technique to help you draw $y = x$, $y = |x|$ and $y = x^2$.

28. Start with $y = x$. Rotate by $90°$. Reflect. Rotate by $-90°$. What do you get?

29. Plot $y = (2x + 1)/2$. Start with $y = x$. Do a stretch. Then shift. Then a shrink. What do you get? Label everything. Show the operations.

30. Plot $y = 3(x + 1)$. Start with $y = x$. Do a shift, then a stretch.

31. Plot $y = 2|x| + 1$. Start with the fundamental curve $y = |x|$. Use the square to help you draw it correctly. Label the axes and show your operations.

32. Plot $y = \dfrac{|x|}{2} - 1$. Start with $y = |x|$. Show all the steps.

33. Plot $y = -2(|x| + 1)$. Start with $y = |x|$. Show all the steps.

34. Plot $y = -(x^2 + 2)$. Start with $y = x^2$. Do a shift, then reflect.

35. Plot $y = -x^2/2$. Start with the parabola $y = x^2$. Do a shrink and then a reflect. Show all the steps and label everything.

36. Plot $y = -2x^2 + 1$. Start with $y = x^2$. Do a stretch. Then a reflect. Finally do a shift. Show all the steps. Label everything and show the operations that you are doing.

7 Transformations and operations

37. Fill in the table.

Transformation	Operation
$y \to y - 1$	
$x \to x/2$	Stretch x by factor of 2.
$y \to -y$	
$x \to x + 1$	

38. Fill in the table.

Transformation	Operation
	Reflect x
	Right shift by 1
	Shrink x by factor of 2.
$y \to 2y$	

39. Fill in the table.

Transformation	Operation
$x \to -y, y \to x$	
$x \to y, y \to -x$	
	Rotate by $-180°$
	Rotate by $180°$.

40. Fill in the table.

Transformation	Operation
	Shift up by 1.
	Stretch y by factor of 2.
$x \to -x$	
$x \to x, y \to y$	

41. Plot $y + 1 = x - 1$. Start with $y = x$. Show the transformations and operations that you are doing. Label the axes and the curves in every step.

42. Plot $y + 2 = |x - 1|$. Start with $y = |x|$.

43. Plot $y = |x + 2| - 1$.

44. Plot $y + 2 = (x - 1)^2$. Start with $y = x^2$.

45. Plot $y^2 = -x$. Start with $y = x^2$ and do a $+90°$ rotation.

46. Plot $y^2 = x$. Start with $y = x^2$ and do a $-90°$ rotation.

47. Plot $y/2 = |x + 1|$. Start with $y = |x|$.

48. Plot $2y = (x - 1)^2$. Start with $y = x^2$.

8 The circle

In every problem, label the axes and the curves in every step. Start with the unit circle centered on the origin $(0, 0)$. Clearly show the transformations and the operations that you are doing.

49. Plot $x^2 + (2y)^2 = 1$.

50. Plot $(2x)^2 + (3y)^2 = 1$.

51. Plot $(x/2)^2 + (2y)^2 = 1$.

52. Plot $(x/5)^2 + (y/5)^2 = 1$.

53. Plot $(x + 2)^2 + (y - 2)^2 = 1$.

54. Plot $(-x - 1)^2 + (-y + 1)^2 = 1$. Do it by factoring out $(-1)^2$.

9 Inequalities

Figure out what values of x make the inequalities true. Use the graph method that we learned in class.

55. $x + 2 > 0$.

56. $x + 2 < 0$.

57. $(x + 1)(x - 2) > 0$.

58. $(x + 1)(x - 2) < 0$.

59. $(x + 3)(x + 1)(x - 2) > 0$.

60. $(x + 3)(x + 1)(x - 2) < 0$.

61. $(x + 2)(x + 1)(x - 1)(x - 2) > 0$.

62. $(x + 2)(x + 1)(x - 1)(x - 2) < 0$.